REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY *(Leave blank)*	2. REPORT DATE March 1996	3. REPORT TYPE AND DATES COVERED Professional Paper

4. TITLE AND SUBTITLE ALGEBRAIC REPRESENTATIONS OF LINGUISTIC AND NUMERICAL MODIFICATIONS OF PROBABILITY STATEMENTS AND INFERENCES BASED ON A PRODUCT SPACE CONSTRUCTION	5. FUNDING NUMBERS PR: CD32 PE: 0601152N WU: DN300173
6. AUTHOR(S) I. R. Goodman	

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Naval Command, Control and Ocean Surveillance Center (NCCOSC) RDT&E Division San Diego, CA 92152–5001	8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) Office of Naval Research Independent Research Program (IR) 800 North Quincy Street Arlington, VA 22217–5660	10. SPONSORING/MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT Approved for public release; distribution is unlimited.	12b. DISTRIBUTION CODE

19960404 087

13. ABSTRACT *(Maximum 200 words)*

This paper addresses the problem of obtaining for any given probability space (Ω, B, P) an extension to a probability space (Ω_o, B_o, P_o) such that for any given function $f:[0,1]^n \rightarrow [0,1]$, suitably analytic, and any given finite collection of ordinary events a, b, c, ..., m in sample space B, with possible relations among them, there is an event $\alpha_f = \alpha_f(a, b, c, ..., m)$ in B_o (not dependent upon P) such that for all possible choices of P and a, b, c, ..., m,

$$P_o(\alpha_f) = f(P(a), P(b), P(c), ..., P(m)),$$

provided the right-hand side is well-defined relative to f. It is seen that the above procedure generalizes that of the problem of constructing "conditional event algebras" relative to all conditional probabilities. Applications to problems on data fusion and combination of evidence are also provided.

Published in *Proceedings of 32nd Annual Bayesian Research Conference*, February 17, 1994.

14. SUBJECT TERMS
Mission Area: Command and Control (C^2), Intelligence
    Probability Space
    Data Fusion
	15. NUMBER OF PAGES

	16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT UNCLASSIFIED	18. SECURITY CLASSIFICATION OF THIS PAGE UNCLASSIFIED	19. SECURITY CLASSIFICATION OF ABSTRACT UNCLASSIFIED	20. LIMITATION OF ABSTRACT SAME AS REPORT

21a. NAME OF RESPONSIBLE INDIVIDUAL	21b. TELEPHONE *(include Area Code)*	21c. OFFICE SYMBOL
I. R. Goodman	(619) 553–4014	Code 4221

/

ALGEBRAIC REPRESENTATIONS OF LINGUISTIC AND NUMERICAL MODIFICATIONS OF PROBABILITY STATEMENTS AND INFERENCES BASED ON A PRODUCT SPACE CONSTRUCTION

Dr. I.R. Goodman,

NCCOSC RDTE DIV 4221
53140 SYSTEMS ST ROOM 230
SAN DIEGO, CA 92152-7550

Tel; (619) 553-4014
Fax: (619) 553-5793
Email: goodman@nosc.mil

EXTENDED ABSTRACT

Summary

In brief, this paper addresses the problem of obtaining for any given probability space (Ω, B, P) an extension to a probability space (Ω_0, B_0, P_0) such that for any given function $f:[0,1]^n \to [0,1]$, suitably analytic, and any given finite collection of ordinary events $a,b,c,...,m$ in sample space B, with possible relations among them, there is an event $\alpha_f = \alpha_f(a,b,c,...,m)$ in B_0 (not dependent upon P) such that for all possible choices of P and $a,b,c,...,m$,

$$P_0(\alpha_f) = f(P(a),P(b),P(c),...,P(m)) , \qquad (1)$$

provided the right-hand side is well-defined relative to f. It is seen that the above procedure generalizes that of the problem of constructing "conditional event algebras" relative to all conditional probabilities. Applications to problems of data fusion and combination of evidence are also provided.

Motivations and More Details

Numerical modifications of individual probability expressions arise quite often based solely on probability evaluations themselves, as. e.g., (1), in the expansion $f(P(a \bullet b), P(b)) = P(a \vee b') = 1 - P(b) + P(a \bullet b)$ or, (2), when a and b are P-independent, $f(P(a), P(b)) = P(a \bullet b) = P(a) \cdot P(b)$, where we note the first expression is an affine combination of $P(b)$ and $P(a \bullet b)$, while the second expression is a an arithmetic product of $P(a)$ and $P(b)$. They can also arise when: (3), we wish to obtain weighted averages of calculated probabilities as in , $f(P(a), P(b)) = w \cdot P(a) + (1-w) \cdot P(b)$, or perhaps (4), as an arithmetic division of $P(a \bullet b)$ by $P(b)$, as in $f(P(a \bullet b), P(b)) = P(a \bullet b)/P(b)$, which of course we recognize as $P(a|b)$, the

conditional probability of a given b. Further modifications to previously calculated probabilities can occur when: (5), linguistic changes are imposed as in the modeling of (meta) statements such as s = P(b), where b = "It is very probable that a" , where a = "John will arrive tomorrow" and where by prior knowledge we can estimate P(a). Also, by prior knowledge, we conclude that the hedge "very probable" typically produces an exponentiation, of say 1.5, so that we obtain the estimate $f(P(a)) = P(a)^{1.5}$, quite reminiscent of the well-known use of exponential or other hedges in fuzzy set modeling. Similarly, (6), we may wish to obtain more complicated compounds of known probabilities in modeling empirically outputs of physical systems as, e.g., in the relation $f(P(a),P(a \bullet b),P(b)) = P(a)^{2.6} + ((3.2) \cdot P(b) \cdot P(a \bullet b)/(4 \cdot P(b) + P(a)))$.

Except for the first two examples, in general for each of the remaining examples 3-6, there is *no* corresponding event α_f, some function (boolean or otherwise) of a,b,c,.. in the original sample space B of the basic events a,b,c,... such that equation (1) is satisfied.

Thus, in general we cannot obtain probability evaluations of further logical combinations of expressions whose modified probability evaluations are known as in (1), such as in the obtaining of

$$P(\alpha \vee \beta) = P(\alpha) + P(\beta) - P(\alpha \bullet \beta) = s + t - P(\alpha \bullet \beta), \qquad (2)$$

where analogous to (1)

$$P(\alpha) = s \quad , \quad P(\beta) = t \quad , \qquad (3)$$

s,t known functions of P(a),P(b),P(a•b),P(c), etc.

On the other hand, a remedy has been developed for the type of problem arising in the fourth example: arithmetic division of probabilities resulting in conditional probabilities can now be treated via some choice of a conditional event algebra (out of many possible candidates).Even though in this case α and β in eq.(3) (where here s and t are arithmetic divisions representing conditional probabilities) do not lie in B, they do lie in a space naturally extending B - the conditional event algebra of choice. (See [1] for background and more details.)

In fact, a recent result [2] has indicated that the *only* mathematically desirable choice for a conditional event algebra is the one arising uniquely from the construction of the product space that has a countable infinity of independent marginal probability spaces, each identical to the one representing the original sample space of ordinary unconditional events [3]. In that space, typically, conditional events as α_f satisfying eq.(1) for Example 4 , where

$$f(P(a \bullet b),P(b)) = P(a \bullet b)/P(b) = P(a|b), \qquad (4)$$

take the form of the natural disjoint cartesian product counterpart to the expansion of arithmetic division in terms of an infinite power series:

$$\alpha_f = (a|b) = \overset{+\infty}{\underset{j=0}{\vee}} \quad (b' \times ... \times b') \times (a \bullet b) \times \Omega \times \Omega \times ..., \qquad (5)$$
$$\text{(j factors)}$$

where Ω is the universal event in sample space (sigma-algebra) B. Thus, for any given probability measure P over B, letting P_0 denote the corresponding product space probability

2

measure extending P, it follows that P_0 applied to a in (5) produces eq.(4), where P is identified with P_0.

It is the contention of this paper that a similar product space construction can be used to develop an algebra of numerical and linguistic modifications of probability statements. This is accomplished by generalizing the forms of α_f in eq.(5), relative to variability of coefficient factors and number of arguments. For example, utilize first the formal relation

$$a^{1/2} = a \cdot (1-a')^{-1/2} \tag{6}$$

and the power series expansion of the second factor, producing the formal power series expansion

$$a^{1/2} = \sum_{j=0}^{+\infty} (a')^j \cdot a \cdot c_j ; \quad c_j =(\text{def}) \binom{2j-1}{j} \cdot 2^{-(2j-1)} , j=0,1,2,.. \tag{7}$$

In turn, this yields the corresponding product space form

$$a^{1/2} = \mathop{v}_{\substack{j=0 \\ (j\ \text{factors})}}^{+\infty} a' \times \ldots \times a' \times a \times \gamma(c_j) \ , \tag{8}$$

where $\gamma(c_j)$ is a conditional event such that for all probability measures P over B,

$$P_0(\gamma(c_j)) = c_j . \tag{9}$$

As an example of such *nontrivial constant-valued events*, consider $\gamma(1/3)$, representing 1/3 in

probability for all P as

$$\gamma(1/3) =(\text{def}) (\ b \times b \times b' \mid b \times b \times b' \ v \ b \times b' \times b \ v \ b' \times b \times b \ ) , \tag{10}$$

where $b \neq \varnothing$, Ω is any event in B - to standardize, preferably in limiting form approaching Ω, whence

$$P_0(\ \gamma(1/3)) = P(b)P(b)P(b') / (P(b)P(b)P(b') + P(b)P(b')P(b) + P(b')P(b)P(b))$$

$$= P(b)^2 P(b') / 3P(b)^2 P(b')$$

$$=1/3. \tag{11}$$

Finally, note that that $a^{1/2}$ is the appropriate appellation for the right-hand side of (8), since P_0 as a product probability measure yields in combination with the disjointness of the terms in eq.(8):

$$P_0(a^{1/2}) = \sum_{j=0}^{+\infty} P(a')^j \cdot P(a) \cdot P(\gamma(c_j)) = P(a) \cdot \sum_{j=0}^{+\infty} P(a')^j \cdot c_j = P(a) \cdot (1-P(a'))^{-1/2} = P(a)^{1/2}. \quad (12)$$

References

1. I.R. Goodman, H.T. Nguyen & E.A. Walker, *Conditional Inference and Logic for Intelligent Systems: A Theory of Measure-Free Conditioning,* North-Holland, Amsterdam, 1991.

2. I.R. Goodman, "A new characterization of product space algebra through deductions", to be submitted.

3. I.R. Goodman & H.T. Nguyen, "A theory of conditional information for probabilistic inference in intelligent systems: Part II, product space approach & Part III, mathematical appendix", to appear in *Information Sciences.*

/